University of California, Berkeley
Physics 110B Fall 2004 (*Strovink*)

FINAL EXAMINATION

Directions: Do all six problems, which have unequal weight. This is a closed-book closed-note exam except for Griffiths, Pedrotti, a copy of anything posted on the course web site, and anything in your own original handwriting (not Xeroxed). A table of spherical harmonics is included with the exam. Calculators are not needed, but you may use one if you wish. Laptops and palmtops should be turned off. Use a bluebook. Do not use scratch paper – otherwise you risk losing part credit. Show all your work. Cross out rather than erase any work that you wish the grader to ignore. Justify what you do. Express your answer in terms of the quantities specified in the problem. Box or circle your answer.

Problem 1. (25 points)
Write down the (real) electric and magnetic fields for a monochromatic plane wave in vacuum of amplitude E_0, angular frequency ω, and phase angle 0 (relative to a cosine). Do it for two cases: the wave is

(a.) (10 points)
traveling in the negative-x direction and polarized in the z direction;

(b.) (15 points)
traveling in the direction from the origin to the point (1,1,0), with polarization perpendicular to the x axis.

Problem 2. (30 points)
Event A occurs at spacetime point $(ct, x, y, z) = (0, 1, 1, 0)$; event B occurs at $(2, 0, 0, 0)$, both in an inertial system $\mathcal{S}$.

(a.) (10 points)
Is there an inertial system $\mathcal{S}'$ in which events A and B occur simultaneously? If so, find $|\vec{r}'_A - \vec{r}'_B|$, the distance in $\mathcal{S}'$ between the two events.

(b.) (10 points)
Is there an inertial system $\mathcal{S}''$ in which events A and B occur at the same spatial coordinates? If so, find $c|t''_A - t''_B|$, c times the magnitude of the time interval in $\mathcal{S}''$ between the two events.

(c.) (10 points)
Can event A be the cause of event B, or vice versa? Explain.

Problem 3. (35 points)
A saucer pilot flying parallel to the ground at velocity $c/\sqrt{2}$ zaps a human (at rest on the ground) with her laser gun. The human observes a pulse of light with wavelength λ_0 incident upon him from directly overhead ($\perp$ to the ground).

(a.) (10 points)
In a convenient coordinate system ($\hat{z}$ up, $\hat{x}$ along the saucer velocity) evaluate the four-vector $k = (\omega/c, \vec{k})$ describing the light pulse $E \propto \cos(\vec{k} \cdot \vec{r} - \omega t)$ seen by the human. Express k in terms of λ_0. Neglect refractive effects in the atmosphere.

(b.) (10 points)
Find the frequency ω' of the light pulse seen by the pilot. (If you wish, you may use the fact that k obeys the Lorentz transformation.)

(c.) (15 points)
At what angle with respect to the vertical direction did the pilot aim her laser?

Problem 4. (35 points)
At $t = 0$, charge $+e$ lies at $(x, y, z) = (0, 0, b/2)$ and charge $-e$ lies at $(x, y, z) = (0, 0, -b/2)$.

(a.) (10 points)
Identify the lowest-l nonvanishing electrostatic multipole moment(s) of the charge distribution (you don't need to calculate its (their) magnitude).

(b.) (10 points)
The static charge distribution in (a.) now is set into oscillation: as time advances, the position vector of each charge is multiplied by the same

factor $1 + \epsilon \cos\omega t$, where ω and $0 < \epsilon \ll 1$ are real constants. Using the fact that a static electric multipole corresponding to a given l and m, when caused to oscillate, yields E-type (TM) multipole radiation of the *same* l and m, what type(s) of lowest-l radiation (*e.g.* $\mathrm{E2}_1$) is (are) emitted?

(c.) (15 points)
For E-type (TM) radiation of type $\mathrm{E}l_m$, the magnetic field $\vec{\tilde{B}}$ ($\perp \hat{r}$) is proportional to the vector spherical harmonic $\vec{X}_{lm}$:

$$\vec{\tilde{B}} \propto \vec{X}_{lm}(\theta, \phi) \equiv \vec{L}\, Y_{lm}(\theta, \phi) \; .$$

Also, in the far zone,

$$\vec{\tilde{E}} \approx c\vec{\tilde{B}} \times \hat{r} \; .$$

Finally, in spherical polar coordinates,

$$i\vec{L} \equiv \vec{r} \times \nabla = \hat{\phi}\frac{\partial}{\partial\theta} - \frac{\hat{\theta}}{\sin\theta}\frac{\partial}{\partial\phi} \; .$$

Write down a function $f(\theta, \phi)$ such that the angular distribution of the radiated power P in the far zone $b \ll \frac{2\pi c}{\omega} \ll r$ is proportional to it:

$$\frac{dP}{d\Omega} \propto f(\theta, \phi) \; .$$

Problem 5. (40 points)
A monochromatic beam of completely unpolarized (natural) light propagates in the $\hat{z}$ direction with initial irradiance I_0. (Natural light can be considered to be the **incoherent** sum of a fully $\hat{x}$ polarized beam and a fully $\hat{y}$ polarized beam.)

The beam passes first through a linear polarizer with transmission direction at an angle ϕ to the $\hat{x}$ axis. Next, it passes through a transparent dielectric slab. The slab is oriented obliquely at the Brewster angle, so that $\hat{y}$ polarized light is fully transmitted. However, the slab transmits only $1/\sqrt{2}$ of the electric field **amplitude** of $\hat{x}$ polarized light.

Downstream of the slab, denote the final irradiance by I_T, and the irradiance transmission ratio by $\mathcal{R} \equiv I_T/I_0$. The quantities I_T and $\mathcal{R}$ are *not supplied by this problem*, *i.e.* your answers should not be expressed as functions of them.

(a.) (10 points)
Suppose (for this part only) that $\phi = 0$. Calculate $\mathcal{R}$.

(b.) (10 points)
Instead suppose (for this part only) that $\phi = 45°$. Calculate $\mathcal{R}$.

(c.) (20 points)
Instead suppose (for the remainder of this problem) that the initial linear polarizer is removed altogether. Calculate the degree of polarization V of the transmitted beam. Recall that

$$V \equiv \frac{I_p}{I_p + I_n} \; ,$$

where I_p is the irradiance of the fully polarized component of the transmitted beam, and I_n is the irradiance of the completely unpolarized component of the transmitted beam.

Hint: Write the (4×1) Stokes vector $\mathcal{S}_0$ of the incident beam as the sum of a Stokes vector $\mathcal{S}_x$ representing fully $\hat{x}$ polarized light and a Stokes vector $\mathcal{S}_y$ representing fully $\hat{y}$ polarized light.

Problem 6. (35 points)
A circularly polarized plane wave of wavelength λ traveling in the z direction is normally incident on a double thin slit (separation $\Delta x = d$). In front of the top slit is placed a quarter wave plate. Obtain the Fraunhofer diffraction pattern $I(\theta)/I(\theta = 0)$, where θ is the angle in the x-z plane. Take the optical thickness of the plate to be such that the irradiance is largest at $\theta = 0$.